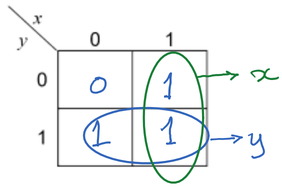
Karnough Maps : a method for simplifying Boolean expressions.

$$\overset{01}{\bar{x}y} + \overset{10}{x\bar{y}} + \overset{11}{xy}$$

$S = \bar{x}y + x\bar{y} + xy = \sum m(1,2,3)$

$S_{optimal} = x + y$

	x	y	$\bar{x}$	$\bar{y}$	$\bar{x}y$	$x\bar{y}$	xy	S
0	0	0						0
1	0	1						1
2	1	0						1
3	1	1						1



Looping rule :

1) Loop adjacent cells
   going from one to the other only one variable change
2) Loop length = power of 2
3) Be greedy (as big of a loop as possible)
4) No redundancy

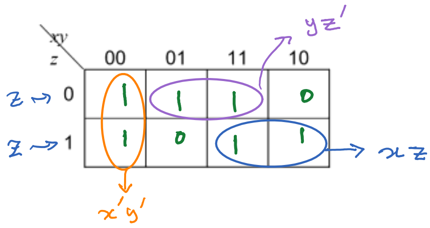
Given

$F = x'y'z' + x'y'z + x'yz' + xy'z + xyz' + xyz$

$\quad\;\; 000 \quad\; 001 \quad\; 010 \quad\; 101 \quad\; 110 \quad\; 111$

	x	y	z	F	
0	0	0	0	1	Col 1
1	0	0	1	1	
2	0	1	0	1	Col 2
3	0	1	1	0	
4	1	0	0	0	Col 4
5	1	0	1	1	
6	1	1	0	1	Col 3
7	1	1	1	1	

$F = \sum m(0,1,2,5,6,7)$



$z \backslash xy$	00	01	11	10
0	0	2	6	4
1	1	3	7	5

$F_{optimal\ sop} = xz + yz' + x'y'$

Simplify the following: $S = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC + A\bar{B}C$ Sop

001 010 011 111 101

$S = \sum m(1, 2, 3, 5, 7)$

C \ AB	00	01	11	10
0	0	1	0	0
1	1	1	1	1

→ C

$\bar{A}B$

Sop optimal form of $S = \bar{A}.B + C$

$\overline{A\bar{C}} = \bar{A} + C$

C \ AB	00	01	11	10
0	0	1	0	0
1	1	1	1	1

$\overline{\bar{B}\bar{C}} = (B + C)$

Pos optimal form

$S = (\bar{A} + C).(B + C)$

001 010 011 111

Simplify the following: $S = \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$ $S = \sum m(1, 2, 3, 7)$

C \ AB	00	01	11	10
0	0	1	0	0
1	1	1	1	0

$A'C$

$A'B$

BC

$S = A'C + A'B + BC$

$F = \bar x \bar y z + \bar x y z + \bar x y \bar z + xyz$

xy / z	00	01	11	10
0	0	1	0	0
1	1	1	1	0

$\bar x y \bar z$

$\bar x \bar y z$ $\quad \bar x y z$ $\quad xyz$

xy / z	00	01	11	10
0	0	1	0	0
1	1	1	1	0

$F = \bar x \bar y z + \bar x y + xyz$

xy / z	00	01	11	10
0	0	1	0	0
1	1	1	1	0

$F = \bar x z + yz + \bar x y$

Simplify the following: $F = \overline{A}\overline{B}C + A\overline{B}\overline{C} + A\overline{B}C$ SOP

001 100 101

$F = \sum m(1, 4, 5)$

AB / C	00	01	11	10
0	0	0	0	1
1	1	0	0	1

Sop optimal $= A\bar B + \bar B C$

	A x	B y	C z	F
0	0	0	0	0
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	1
5	1	0	1	1
6	1	1	0	0
7	1	1	1	0

$\overline{\bar A \bar C} = A + C$

$\bar B$

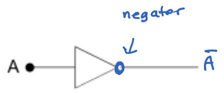
AB / C	00	01	11	10
0	0	0	0	1
1	1	0	0	1

POS optimal ~~$F = (A+C) + \bar B$~~

$F = (A+C) \cdot \bar B$

Logic Gates

Negator



only one input

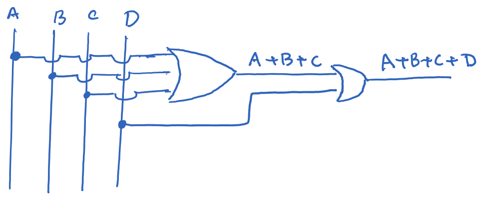
OR

(2 or 3 inputs only)

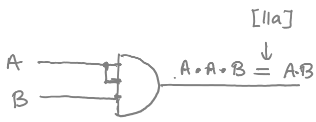
AND

(2 or 3 inputs only)

Make A + B + C + D (use 2 or 3 inputs only)



Make A · B by using 3-input and gate.



Word Problems

1. In the old days when computers were new, it cost 15¢ to buy a cup of coffee from a vending machine. The machine has three coin slots, one of which accepts a quarter, one a dime, and a third a nickel. Each slot sends a signal to the dispenser if the slot contains a coin when the coin holder is pressed in. The goal is to design a circuit which will output a 'T' signal if at least 15¢ has been collected.

Modelling assumptions: at most one type of each coin is deposited and the machine doesn't give change back.

Assumptions :

- At most one coin of each type
- No change is given back

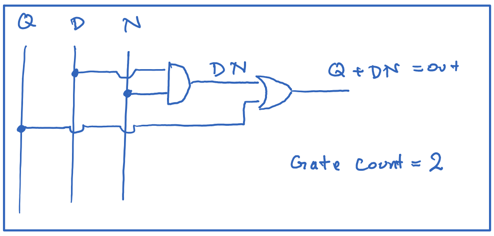
	Q	D	N	out
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

N \ QD	00	01	11	10
0	0	0	1	1
1	0	1	1	1

DN

Q

$$Out = Q + DN$$



2. A beverage machine dispenses two kinds of coffee (say type A and type B) and cream. Let

A = 1 when there is type A coffee to dispense
B = 1 when there is type B coffee to dispense
C = 1 when there is cream available to dispense

Develop a logical function, X, which returns 1 whenever at least one kind of coffee is available to dispense, but returns zero if no coffee or nothing at all is available to dispense.

	A	B	C	X
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	1
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

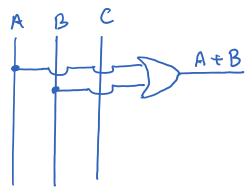
C \ AB	00	01	11	10
0	0	1	1	1
1	0	1	1	1

→ A

B

$X = \sum m(2,3,4,5,6,7)$

$X = A + B$



A B C

3. Three identical control units are incorporated into a critical unit of some facility, say. Each control unit returns a high (or 1) signal if it is in a ready state. Develop a logical function which will itself return a value 1 whenever two or more of these redundant control units are not in a ready state.

A , B , C → 0 not ready
→ 1 is ready

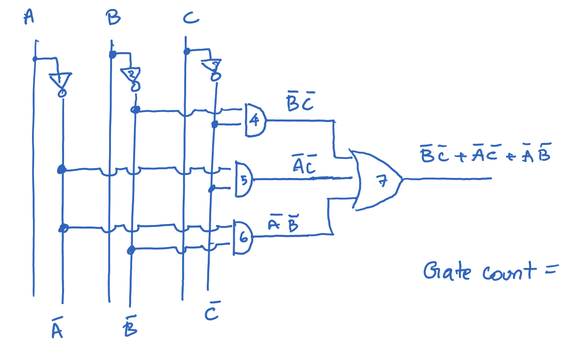
out → 0
→ 1 if two or three of A, B, C = 0

	A	B	C	out	
0	0	0	0	1	1
1	0	0	1	1	
2	0	1	0	1	2
3	0	1	1	0	
4	1	0	0	1	4
5	1	0	1	0	
6	1	1	0	0	3
7	1	1	1	0	

C \ AB	00	01	11	10
0	1	1	0	1
1	1	0	0	0

Sop

$out = \bar{B}\bar{C} + \bar{A}\bar{C} + \bar{A}\bar{B}$



Gate count = 7

Canonical and optimal forms

Canonical: made of and, or, not

optimal: as simple as possible

Four-variable Karnaugh Map

cell numbering:

zu \ xy	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

ex 1

$F = \prod M(\underbrace{0,3,7,8,9,10,12,14}_{\text{where the zeros are}})$

zu \ xy	00	01	11	10
00	0	1	0	0
01	1	1	1	0
11	0	0	1	1
10	1	1	0	0

sop → loop 1's

pos → loop 0's

$F = \bar x \bar z u + \bar x z \bar u + x z u + x y u + \bar x y \bar z$ // optimal sop

ex 2:

Find pos optimal

zu \ xy	00	01	11	10
00	1	0	0	1
01	0	1	1	0
11	0	1	1	0
10	1	1	0	1

$\overline{xy\bar u} = (\bar x + \bar y + u)$

$\overline{y \bar z \bar u} = (\bar y + z + u)$

$\overline{\bar y u} = (y + \bar u)$

$Out = (\bar x + \bar y + u) \cdot (\bar y + z + u) \cdot (y + \bar u)$

ex 3

zu \ xy	00	01	11	10
00	1	0	0	1
01	0	0	1	0
11	0	0	1	0
10	1	1	0	1

pos

$\overline{\bar x u} = (x + \bar u)$

$\overline{x y \bar u} = (\bar x + \bar y + u)$

$\overline{\bar y u} = (y + \bar u)$

$\overline{y \bar z \bar u} = (\bar y + z + u)$

$F = (\bar y + z + u) \cdot (x + \bar u) \cdot (\bar x + \bar y + u) \cdot (y + \bar u)$

pos optimal

44

Example: Simplify the Boolean function given by $Q(x,y,z,u) = \sum m(3,4,5,6,11,12,13,14)$

row #	x	y	z	u	Q
0					0
1					0
2					0
3					1
4					1
5					1
6					1
7					0
8					0
9					0
10					0
11					1
12					1
13					1
14					1
15					0

zu \ xy	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	1	0	0	1
10	0	1	1	0

$Q = yu' + y'zu + yz'$ = SOP optimal

Example: Simplify the Boolean function given by $F(A,B,C,D) = \sum m(3,6,7,10,11,13,14,15)$

CD \ AB	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	1	1	1	1
10	0	1	1	1

In SOP form

$F = CD + ABD + AC + BC$

Example: Simplify the Boolean function given by $F(A,B,C,D) = \prod M(0,2,3,4,8,9,10,14)$

location of 0's

Find optimal POS :

CD \ AB	00	01	11	10
00	0	0	1	0
01	1	1	1	0
11	0	1	1	1
10	0	1	0	0

$A\bar{B}$

$\overline{A\bar{B}\bar{C}}$ $\overline{\bar{A}\bar{C}\bar{D}}$ $\overline{\bar{A}\bar{B}C}$ $\overline{AC\bar{D}}$

$F = (\bar{A}+B+C)\cdot(A+C+D)\cdot(A+B+\bar{C})\cdot(\bar{A}+\bar{C}+D)$

Example: Simplify the Boolean function given by $F = a\bar{b}c\bar{d} + a\bar{b}\bar{c}\bar{d} + a\bar{b}cd + a\bar{b}\bar{c}d + \bar{a}b\bar{c}d$

$1010 = 10 \quad 1000 = 8 \quad 1011 = 11 \quad 1001 = 9 \quad 0101 = 5$

$F = \sum m(5, 8, 9, 10, 11)$

in Sop format

Sop

cd \ ab	$\bar{a}\bar{b}$ 00	$\bar{a}b$ 01	ab 11	$a\bar{b}$ 10
$\bar{c}\bar{d}$ 00	0	0	0	1
$\bar{c}d$ 01	0	1	0	1
cd 11	0	0	0	1
$c\bar{d}$ 10	0	0	0	1

$F = a\bar{b} + \bar{a}b\bar{c}d$

Example: Simplify the Boolean function given by $F = abd + b\bar{c}\bar{d} + c\bar{d}$

Sop

a b c d

$1101 \quad 0100$
$1111 \quad 1100$

0010	2	4	
0110	6	12	
1010	10	13	
1110	14	15	

$F = \sum m(2, 4, 6, 10, 12, 13, 14, 15)$

cd \ ab	00	01	11	10
00	0	1	1	0
01	0	0	1	0
11	0	0	1	0
10	1	1	1	1

$F = ab + cd' + bd'$